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Use of the Laplace Transformation for Data Reduction in the Flash Method of Measuring Thermal Diffusivity

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This paper presents a new way to reduce data in the laser flash method of measuring thermal diffusivity. Experimental temperature vs time data are first transformed by using the Laplace transformation, and then they are fitted with an appropriate theoretical formula. The data reduction procedure is more efficient and enables the use of more realistic models of heat conduction in the sample, because the theoretical formulae for transformed temperatures have a simpler form than those for nontransformed ones. Some examples of the theoretical formulae of transformed temperatures are included here for one- and two-dimensional heat transfer, respectively. The models described take into account a finite pulse time and heat losses from the sample. Two fitting algorithms are proposed. Experimentally, the data reduction procedure has been tested for a correction of the finite pulse time effect in the flash method. The results show that the accuracy of our procedure is comparable with other data reduction methods. Provided that the shape and duration of the pulse are known, this procedure allows elimination of the finite pulse time effect on calculation of the thermal diffusivity for any transformable heat pulse time function, even in cases where the other specialized data reduction procedures have failed.

KEY WORDS: flash method; heat conduction equation; Laplace transformation; thermal diffusivity.

1. INTRODUCTION

This paper presents a new way to reduce data in the laser flash method of measuring thermal diffusivity [1]. In this method, the front surface of a

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small disk-shaped sample receives a pulse of radiant energy from a laser, and the resulting temperature response on the opposite (rear) surface of the sample is recorded. The existing data reduction procedures for computing the desired value of thermal diffusivity are based on fitting the experimental temperature–time data with the theoretically computed curve. Due to the complexity of the theoretical formulae, as well as the problems with the exact analytical formulation, only relatively simple theoretical models can be taken into account in these procedures. The most simple (ideal) model assumes that the sample is isotropic, homogeneous, opaque to the pulse radiation, and thermally insulated and that the heat pulse is instantaneous and uniformly absorbed in the front surface of the sample. Using a more realistic model is complicated by the fact that analytical formulae for temperature curves are commonly very complex, and their form depends strongly on the assumed model. Existing fitting procedures are made using special boundary and initial condition. They differ from case to case, with various limitations imposed on their applicability due to the approximations used in the analytical formulae.

The new method for data reduction is based on the well-known fact that the Laplace transforms of the analytical temperature vs time formulae have simpler forms than the original ones. Curve-fitting procedures of the transformed experimental data with the theoretical ones are more efficient and faster than those treating the nontransformed data.

Although a similar approach based on Laplace transform of temperature vs time data has been used previously in other methods, this is, as far as we know, the first application to the flash method. Iida et al. [2–4] proposed three simple methods for measuring the thermal diffusivity of solids using a Laplace transformation of the sample temperature. The sample in these methods has the form of a circular cylinder [2, 3] or a flat plate [4], and the temperature response from a heat input is recorded for two positions. Delpech et al. [5] described the use of the Laplace transform for data reduction in the so-called "front face" flash method for measuring thermal properties. In this technique, the temperature response caused by the heat pulse is monitored on the irradiated front face of a sample. In addition to the difference between the experimental setups, their algorithms for computing the thermal diffusivity are completely different from those proposed here.

In this paper, a brief mathematical background in using the Laplace transformation in data reduction for the flash method is given. Some examples of the theoretical formulae for the transformed temperature are included here for one- and two-dimensional heat transfer. The models described take into account finite pulse time and heat losses from the sample. As a demonstration of the feasibility of this method of data reduction, the problem of finite pulse time in flash method is discussed in detail for the case of a triangular shape pulse. The proposed data reduction procedure was tested on an experimental set of temperature curves.

2. THEORETICAL MODELS

The theoretical formulae needed for the data reduction procedure can be derived as a solution of the heat conduction equation, along with initial and boundary conditions which correspond to the chosen model. In the case where the heat flux in the sample can be treated as one dimensional, then the heat conduction equation is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad (0 \leqslant x \leqslant L, \qquad t > 0) \tag{1}$$

where t is time, α thermal diffusivity, x the space coordinate, L the sample thickness, and $T = T(x, t)$ the temperature at the space-time point (x, t). In the above-mentioned ideal model, the conditions are

$$T(x, 0) = 0 \tag{2}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = -\frac{Q}{k}\,\delta(t), \qquad \text{and} \qquad \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \tag{3}$$

where k is the thermal conductivity of the sample, Q is the amount of heat absorbed through unit surface of the sample, and $\delta(t)$ is the Dirac delta function.

The solution of Eq. (1), with the conditions given by Eqs. (2) and (3) for $x = L$, and $t > 0$, can be expressed by the known formula [1]

$$T(L, t) = T_0 \left[1 + 2 \sum_{n=0}^{\infty} (-1)^n \exp\left(-\frac{\pi^2 n^2 \alpha t}{L^2} \right) \right] \tag{4}$$

where

$$T_0 = \frac{Q}{\rho c L} \tag{5}$$

is the steady-state temperature in the sample after the pulse, ρ is the density, and c is the heat capacity of the sample material.

The Laplace transform $\bar{v}(p)$ of a function $v(t)$ is defined as

$$\bar{v}(p) = \int_0^\infty e^{-pt} v(t)\, dt \tag{6}$$

where p is a number whose real part is positive and large enough to make the where p is a number whose real part is positive and large enough to make the integral given by Eq. (6) convergent [6]. There are at least two ways to compute the theoretical formula for the Laplace transform of the temperature in the sample. The first is to insert the known expression for $T[L, t)$ into Eq. (6) and to find a solution of this integral. The second, more convenient way is to make a Laplace transformation of the heat conduction equation with the initial and the boundary conditions and to solve this differential equation with the transformed boundary conditions for the unknown function $\bar{T}(L, p)$.

In the case of the ideal model, the Laplace transform of the temperature on the rear surface of the sample after a pulse has the form

$$\bar{T}(L, p) = T_0 \frac{L}{\sqrt{\alpha p}} \sinh^{-1}\left(\sqrt{\frac{p}{\alpha}}L\right) \tag{7}$$

A comparison of Eq. (4) with Eq. (7) reveals that the Laplace transform is a simpler function, more suitable for a numerical or analytical treatment. As shown below, this is a typical feature of these formulae also in more realistic models.

In the case when the pulse is not instantaneous and the heat flux across the front sample surface is described by a function $f(t)$, the only change in formulation of conditions given by Eqs. (2) and (3) is that the boundary condition for $x = 0$ now has the form

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = -\frac{Q}{k} f(t) \tag{8}$$

The desired Laplace transform of the temperature is, in this case, given by

$$\bar{T}(L, p) = T_0 \frac{L}{\sqrt{\alpha p}} \bar{f}(p) \sinh^{-1}\left(\sqrt{\frac{p}{\alpha}}L\right) \tag{9}$$

where $\bar{f}(p)$ represents the Laplace transform of the heat pulse function $f(t)$. The most usual forms of this function are shown in Table I.

The temperature at the rear surface of the sample after an exponential heat pulse, as derived by Larson and Koyama [7], has the form

$$T(L, t) = T_0 \left\{ 1 + 2\frac{L^4}{(\alpha t_p)^2} \sum_{n=1}^{\infty} (-1)^n \frac{\exp[-(n^2\pi^2\alpha t/L^2)]}{[(L^2/\alpha t_p) - n^2\pi^2]^2} - \frac{(L/\sqrt{\alpha t_p}) \exp[-(t/t_p)]}{2\sin(L/\sqrt{\alpha t_p})} \left[1 + 2\frac{t}{t_p} + \frac{L}{\sqrt{\alpha t_p}} \cot\left(\frac{L}{\sqrt{\alpha t_p}}\right) \right] \right\} \tag{10}$$

Table I. Heat Pulse Time Distribution Functions and Their Laplace Transforms

Heat pulse	$f(t)$		$\bar{f}(p)$
Instantaneous (Dirac delta function)	$\delta(t)$		1
Square (μ Heaviside unit step function; τ duration)	$\dfrac{\mu(t)-\mu(t-\tau)}{\tau}$		$\dfrac{1-\exp(-p\tau)}{p\tau}$
Triangular (t_v time of maximum; τ duration)	$\dfrac{2}{\tau}\dfrac{t}{t_v}$	$(0 \leqslant t \leqslant t_v)$	$\dfrac{2}{\tau t_v p^2}\left[1-\dfrac{\tau e^{-pt_v}-t_v e^{-p\tau}}{\tau - t_v}\right]$
	$\dfrac{2}{\tau}\dfrac{\tau-t}{\tau-t_v}$	$(t_v \leqslant t \leqslant \tau)$	
	0	$(0 \geqslant t \geqslant \tau)$	
Exponential (t_p time of maximum)	$\dfrac{t}{t_p^2}\exp\left(-\dfrac{t}{t_p}\right)$		$\dfrac{1}{t_p^2(p+t_p^{-1})^2}$

Comparison of Eq. (10) with its Laplace transform, given by

$$\bar{T}(L, p) = T_0 \frac{L}{\sqrt{\alpha p}} \cdot \frac{1}{t_{\mathrm{p}}^2(p+t_{\mathrm{p}}^{-1})^2} \sinh^{-1}\left(\sqrt{\frac{p}{\alpha}}L\right) \tag{11}$$

shows the advantage of using the latter for fitting in a data reduction process.

In the case, where heat is lost from both surfaces of the sample, the boundary conditions given by Eq. (3) have the form

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = -\frac{Q}{k} f(t) + \frac{H_0}{L} T(0, t), \qquad \left.\frac{\partial T}{\partial x}\right|_{x=L} = -\frac{H_L}{L} T(L, t) \tag{12}$$

and the Laplace transform of temperature is now given by

$$\bar{T}(L, p) = T_0 \frac{L}{\sqrt{\alpha p}} \bar{f}(p) \left[\left(1 + \frac{H_0 H_L}{L^2} \frac{\alpha}{p}\right) \sinh\left(\sqrt{\frac{p}{\alpha}} L\right) + \frac{H_0 + H_L}{L} \sqrt{\frac{\alpha}{p}} \cosh\left(\sqrt{\frac{p}{\alpha}} L\right)\right]^{-1} \tag{13}$$

where H_0 and H_L are the Biot numbers for the front and rear sample surface, respectively.

If heat is lost from all surfaces of a disk-shaped sample and an axisymmetrical heat source acts on the front surface of the sample, then the conductive heat transfer is described by a two-dimensional heat conduction equation in cylindrical coordinates (see Fig. 1.)

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] \quad (0 \leqslant x \leqslant L, \quad 0 \leqslant r \leqslant R, \quad t > 0) \tag{14}$$

with boundary and initial conditions

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = -\frac{Q}{k} g(r) f(t) + \frac{H_0}{L} T(0, r, t), \quad \left. \frac{\partial T}{\partial x} \right|_{x=L} = -\frac{H_L}{L} T(L, r, t) \tag{15}$$

$$T(x, 0, t) < \infty, \qquad \left. \frac{\partial T}{\partial r} \right|_{r=R} = -\frac{H_R}{R} T(x, R, t) \tag{16}$$

$$T(x, r, 0) = 0 \tag{17}$$

where function $g(r)$ describes the radial distribution of the heat pulse, and H_R is the Biot number for the lateral surface at $r = R$.

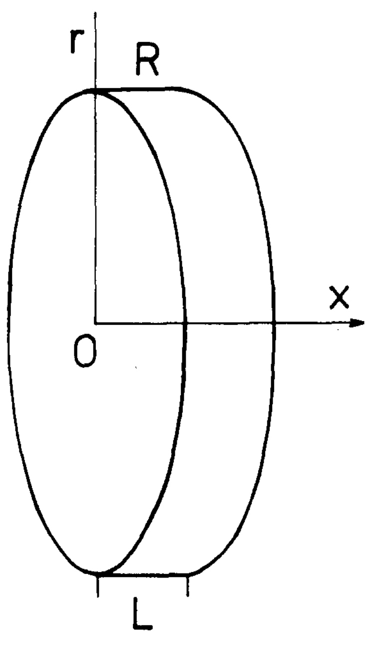

Fig. 1. System of coordinates in the sample.

As shown in the Appendix, the Laplace transform of the temperature rise in the middle of the rear face (at $x = L$, $r = 0$) then has the form

$$\bar{T}(L, 0, p) = T_0 \frac{2L^2}{\alpha R^2} \bar{f}(p) \sum_{i=1}^{\infty} \frac{\mu_i^2 \beta_i \bar{\bar{g}}(\mu_i)]}{J_0^2(\mu_i)(H_R^2 + \mu_i^2) \begin{bmatrix} (\beta_i^2 + H_0 H_L) \sinh(\beta_i) \\ + \beta_i (H_0 + H_L) \cosh(\beta_i) \end{bmatrix}} \tag{18}$$

where μ_i are positive roots of the equation

$$\mu J_1(\mu) - H_R J_0(\mu) = 0 \tag{19}$$

and

$$\beta_i = L \sqrt{\frac{\mu_i^2}{R^2} + \frac{p}{\alpha}}, \qquad \bar{\bar{g}}(\mu_i) = \int_0^R g(r)\, J_0(\mu_i r/R)\, r\, dr \tag{20}$$

The series in Eq. (18) converges quite rapidly so, for practical purposes, only a few terms have to be computed.

A comparison of formulae for the temperature in a multilayer sample (see, e.g., Ref. 8) also shows the relative simplicity of the Laplace-transformed ones.

3. DATA REDUCTION PROCEDURES

Calculation of thermal diffusivity values from the experimental temperature vs time data is the aim of a data reduction procedure in the flash method. The method proposed here differs from presently used procedures in that the experimental data are first transformed by using a Laplace transformation, then fitted to relevant theoretical formula.

Experimental temperature data E_i can be transformed by using the definition integral given by Eq. (6) in the form

$$\bar{E}(p) = \int_0^{t_m} e^{-pt} E_i\, dt \tag{21}$$

where the infinite upper limit for time is replaced by a maximal value of measured time period t_m. The error incurred by replacing the upper limit in Eq. (21) with a finite time depends on the value of the product pt_m. For practical purposes, the number p should be chosen large enough to fulfill the condition $pt_m > 5$. For values $pt_m > 25$, the "later" parts (approximately for time $> t_m/3$) of the temperature vs time curve have practically no contribution to the value of the integral given by Eq. (14).

The integral given by Eq. (21) can be solved numerically by using Simpson's 1/3 rule [9]:

$$\bar{E}(p) \approx \frac{\Delta t}{3} [E_0 + 4(e^{-p\Delta t}E_1 + e^{-3p\Delta t}E_3 + \cdots + e^{-(N-1)p\Delta t}E_{N-1})$$
$$+ 2(e^{-2p\Delta t}E_2 + e^{-4p\Delta t}E_4 + \cdots + e^{-(N-2)p\Delta t}E_{N-2}) + e^{-Np\Delta t}E_N] \quad (22)$$

where Δt is the time interval between two consecutively recorded values of temperature, E_0 is the temperature at zero time (beginning of the heat pulse), E_1 is the temperature at time Δt, E_2 is the temperature at time $2\Delta t$, and so on. The last value, E_N, corresponds to temperature at time $t_m = N\,\Delta t$, where N is an even integer number.

3.1. Two-Point Algorithm

There exists a wide variety of curve-fitting algorithms that can be applied to the procedure for finding the desired thermal diffusivity as a parameter involved in $\bar{T}(L, p)$. As can be seen from Eq. (9), (13), or (18), the second unknown parameter—steady-state temperature after the pulse, T_0—can be eliminated by dividing two values of the transformed temperature calculated for different values of p. Comparison of the rates of the transformed experimental data with the theoretical one, where a suitable formula for $\bar{T}(L, p)$ is used, is the basis for the so-called "two-point" algorithm. The function F, defined as

$$F(\alpha) = \frac{\bar{E}(p_1)}{\bar{E}(p_2)} - \frac{\bar{T}(L, p_1)}{\bar{T}(L, p_2)} \quad (23)$$

has one simple root, α^*, which is the desired thermal diffusivity of the sample. The problem of finding α reduces to solving an equation $F(\alpha) = 0$.

3.2. Least-Squares Algorithm

Another way to fit the transformed experimental data to a theoretical curve, which is highly recommended for a noisy signal, is to use a standard least-squares method of finding the optimal values of the unknown parameters. The algorithm for nontransformed data, described in Ref. 10, can be easily applied to the transformed ones. In this algorithm, which enables us to find both the value of the thermal diffusivity and the steady-state temperature of the sample, we seek a minimum of the least-squares function

$$R(\alpha, T_0) = \sum_{j=1}^{K} \{\bar{E}(p_j) - \bar{T}(L, p_j)\}^2 \quad (24)$$

where $\bar{E}(p_j)$ is a Laplace transformation of experimental temperature data calculated for the parameter p_j, with $j = 1, 2, 3, \ldots, K$, and $\bar{T}(L, p_j)$ stands for a suitable transformed theoretical temperature function [for example, that given by Eq. (9), (13) or (18)]. The necessary conditions for the extremes of $R(\alpha, T_0)$ are given by the set of equations

$$\frac{\partial R(\alpha, T_0)}{\partial \alpha} = 0, \qquad \frac{\partial R(\alpha, T_0)}{\partial T_0} = 0 \tag{25}$$

After performing the indicated operations, the equations for both parameters can be obtained in the form

$$\sum_{j=1}^{K} \bar{E}_j \bar{T}_j(\alpha) \sum_{j=1}^{K} \bar{T}_j(\alpha) \frac{\partial \bar{T}_j(\alpha)}{\partial \alpha} - \sum_{j=1}^{K} \bar{E}_j \frac{\partial \bar{T}_j(\alpha)}{\partial \alpha} \sum_{j=1}^{K} \bar{T}_j^2(\alpha) = 0 \tag{26}$$

$$T_0 = \sum_{j=1}^{K} \bar{E}_j \bar{T}_j(\alpha) \left[\sum_{j=1}^{K} \bar{T}_j^2(\alpha) \right]^{-1} \tag{27}$$

where, for simplicity, $\bar{E}_j \equiv \bar{E}(p_j)$, and $\bar{T}_j(\alpha) \equiv \bar{T}(L, p_j)/T_0$. Here, as well as in the two-point algorithm, we assume that the only unknown parameters in $\bar{T}(L, p_j)$ are α and T_0.

The optimal value of the parameter α can be found numerically by solving Eq. (26). T_0 can be then calculated directly from Eq. (27). The function on the left-hand side of Eq. (26) depends only on α and has only one simple root α^*, which corresponds to the desired value of thermal diffusivity. The value of T_0 is a useful parameter in some calorimetric applications of the flash method.

4. DISCUSSION

To demonstrate the use of this data reduction procedure in the flash method, the effect of the finite pulse time (when the duration of the heat pulse has a finite value) on the reproducibility of thermal diffusivity calculations is discussed here in detail. A set of temperature vs time curves $E(t_i)$ was generated by the flash method, in which the front face of a disk-shaped ceramic sample was subjected to a pulse of intense light from a halogen lamp. The pulse shape is representable by a triangle of duration τ with maximum at $\tau/2$, as was determined by independent optical measurement. The curves (see Fig. 2.) differ from one to another due to the duration of the pulse.

The Simpson's 1/3 rule, given by Eq. (22), was used for Laplace transformation of $E(t_i)$, where the parameter p is from 0.5 to 1.6 s^{-1} with step 0.1 s^{-1} and $t_m = 10$ s. The transformed temperature curves are given in Fig. 3.

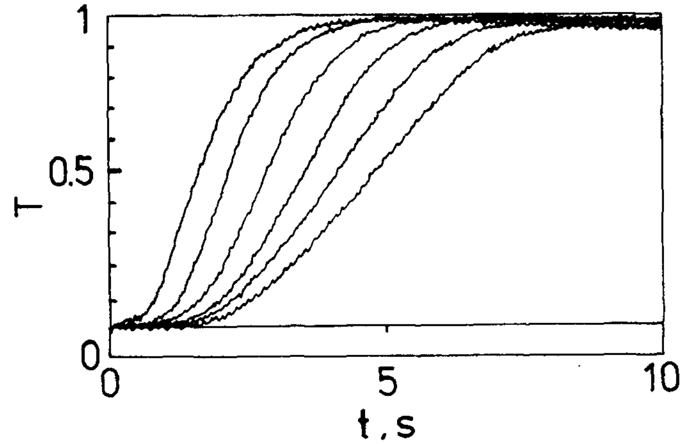

Fig. 2. Temperature vs time curves $E(t)$ with various values of the pulse duration. For the curves from left to right, $\tau = 0.1$, 1.0, 2.0, 3.0, 4.0, and 5.0 s.

For calculating the value α of the thermal diffusivity, the least-squares algorithm (LSA) was applied to the transformed data, using the function

$$\bar{T}_j(\alpha) = \frac{L}{\sqrt{\alpha p_j}} \frac{4}{\tau^2 p^2} \left[1 - 2e^{-p\tau/2} + e^{-p\tau}\right] \sinh^{-1}\left(\sqrt{\frac{p_j}{\alpha}}\, L\right) \tag{28}$$

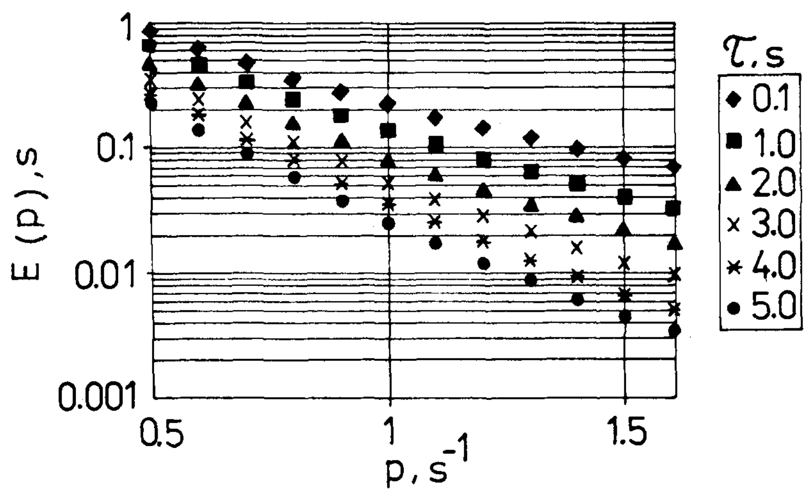

Fig. 3. Transformed temperature vs time curves $E(p)$ with various values of the pulse duration τ.

in Eq. (26). Function $\bar{T}_j(\alpha)$, given in Eq. (28), was derived from Eq. (9), where $\bar{f}(p)$ stands for a triangular heat pulse, and $t_v = \tau/2$. All 12 points for the value of p were taken into account in this curve-fitting procedure. The least-squares algorithm was chosen because the signal (as shown in Fig. 2) is fairly noisy and the curves have to be smoothed before applying the data reduction procedures.

For a comparison of reproducibility as well as accuracy, our algorithm was tested along with the method of data reduction suggested by Azumi and Takahashi [11] and the method proposed by Taylor and Clark [12].

The Azumi and Takahashi method consists of adjusting the time origin for an "effective" irradiation time of the sample, using the temporal center of gravity of the heat pulse t_g, defined as

$$t_s = \int_0^\tau t' f(t')\, dt' \Big/ \int_0^\tau f(t')\, dt' \tag{29}$$

where τ is the duration of the heat pulse, and f is a heat pulse function. In our case $t_s = \tau/2$. Thermal diffusivity α is then calculated from the equation

$$\alpha = 0.1388 \frac{L^2}{t_{1/2} - t_s} \tag{30}$$

where $t_{1/2}$ is the time from the beginning of a pulse until the temperature of the rear face of the sample reaches half of its maximum value.

Taylor and Clark, in their method of correction for the finite pulse time, suggest calculating thermal diffusivity using the equation

$$\alpha = \frac{c_1 L^2}{c_2 t_{1/2} - \tau} \tag{31}$$

where the known constants c_1 and c_2 depend on the shape of the pulse. In our case Eq. (31) is

$$\alpha = \frac{0.27057 L^2}{1.9496 t_{1/2} - \tau} \tag{32}$$

The results of this test are summarized in Table II. Uncorrected values of thermal diffusivity, calculated using the form suggested by Parker et al. [1],

$$\alpha = 0.1388 \frac{L^2}{t_{1/2}}, \tag{33}$$

are given in this table, too.

Because the exact reference value for the thermal diffusivity of measured ceramic material is not known, the evaluation of accuracy of our method is based only on the comparison of results obtained by different

Table II. Results of the Experimental Test[a]

	LSA[b]		Azumi et al. [11]		Taylor et al. [12]		Parker et al. [1]	
τ(s)	α	Δ	α	Δ	α	Δ	α	Δ
0.1	7.775	0	8.059	0	8.065	0	7.808	0
1.0	7.775	0	7.808	−3.1	7.870	−2.4	5.949	−23.8
2.0	8.017	3.3	6.752	−16.2	7.040	−12.7	4.461	−42.8
3.0	7.732	0.6	6.246	−22.5	6.369	−21.0	3.569	−54.3
4.0	7.867	1.2	6.094	−24.4	6.251	−22.5	3.084	−60.5
5.0	7.597	−2.3	5.678	−29.5	5.849	−27.5	2.658	−66.0

[a] Thermal diffusivity α is in $10^7 m^2 \cdot s^{-1}$, and the value Δ is calculated as the relative difference in the percentage of the actual value of α from the first value in the column.
[b] Least-squares algorithm.

data reduction methods. Under the plausible assumption that the temperature curve for $\tau = 0.1$ s is the least affected by the finite pulse width (the duration of the pulse τ is about 20 times smaller than the half-time $t_{1/2}$) and that Azumi's and Taylor-Clark's method can (for this curve) eliminate this effect, then, as shown in the first row in Table II, the accuracy of our method is comparable with the other specialized procedures. The LSA can be successfully applied the other special data reduction procedures already failed (when τ become comparable with $t_{1/2}$). In fact, there are no limitations for using our data reduction procedure for elimination of a finite pulse time effect, provided that the shape and the duration of the pulse are (at least approximately) known.

One of the features of the Laplace transformation, inherent in its nature, is that the initial part (in the sense of time) of an original function is taken into the transformation with a higher "weight" than later parts. This property can be regarded as an advantage from the point of view of the heat loss effect, where the sample is not adiabatically isolated. In this case, the initial part of the temperature vs time curve is less affected by the heat losses from the sample, and it is reasonable to use this part for data reduction.

On the other hand, the very early part is often distorted by direct interaction of the heat pulse radiation with the temperature sensor or by the time response of the sensor, and this region may be practically unusable. One way to minimize this disadvantage is to choose an appropriate frequency of sampling of the experimental data which is large enough to skip the distorted interval.

5. CONCLUSIONS

The present study describes a new way of reducing data in the flash method of measuring thermal diffusivity. Experimental temperature vs time data are first transformed using the Laplace transformation, and the transformed data are then fitted with a suitable theoretical formula. This improves the efficiency and overall performance of the data reduction procedure, as well as permitting the use of more realistic models for a description of heat conduction in the sample. This is due to the fact that the theoretical formulae for the transformed temperature have (as a rule) simpler forms than those for nontransformed cases. Two algorithms for the curve-fitting procedure were proposed, too. Experimentally, the data reduction procedure has been tested on a correction for the finite pulse time effect in the flash method. The results show that the accuracy of our procedure is comparable with that of other data reduction methods. Provided that the shape and duration of the pulse are known, this procedure allows elimination of the finite time effect on calculation of the thermal diffusivity for any transformable heat pulse time function, even in cases where other known data reduction procedures have failed.

APPENDIX

In order to solve Eq. (14) with conditions given by Eqs. (15)–(17), a finite Hankel transformation with regard to r and a Laplace transformation can be used successively.

Writing $\tilde{T}$ for the finite Hankel transform of T, it follows from Eqs. (14)–(17) that $\tilde{T}$ satisfies

$$\frac{1}{\alpha}\frac{\partial \tilde{T}}{\partial t}=\frac{\partial^2 \tilde{T}}{\partial x^2}-\frac{\mu_i^2}{R^2}\tilde{T} \tag{A1}$$

with conditions

$$\left.\frac{\partial \tilde{T}}{\partial x}\right|_{x=0}=-\frac{Q}{k}\,\tilde{g}(\mu_i)\,f(t)+\left.\frac{H_f}{L}\tilde{T}\right|_{x=0},\qquad \left.\frac{\partial \tilde{T}}{\partial x}\right|_{x=L}=-\left.\frac{H_r}{L}\tilde{T}\right|_{x=L} \tag{A2}$$

$$\tilde{T}|_{t=0}=0 \tag{A3}$$

where

$$\tilde{T}=\int_0^R T(x,r,t)\,K(\mu_i,r)\,r\,dr,\qquad \tilde{g}(\mu_i)=\int_0^R g(r)\,K(\mu_i,r)\,r\,dr$$

$$K(\mu_i,r)=\frac{\sqrt{2}\,\mu_i J_0(\mu_i r/R)}{R J_0(\mu_i)\sqrt{(H_s^2+\mu_i^2)}}$$

and μ_i are positive roots of Eq. (19).

Writing $\bar{\bar{T}}$ for the Laplace transform of $\tilde{T}$, it follows from Eqs. (A1)–(A3) that $\bar{\bar{T}}$ satisfies

$$\frac{\partial^2 \bar{\bar{T}}}{\partial x^2} - \left(\frac{\mu_i^2}{R^2} + \frac{p}{\alpha}\right) \bar{\bar{T}} = 0 \tag{A4}$$

with

$$\left.\frac{\partial \bar{\bar{T}}}{\partial x}\right|_{x=0} = -\frac{Q}{k}\,\tilde{g}(\mu_i)\,\bar{f}(p) + \frac{H_\mathrm{f}}{L}\left.\bar{\bar{T}}\right|_{x=0}, \qquad \left.\frac{\partial \bar{\bar{T}}}{\partial x}\right|_{x=L} = -\frac{H_\mathrm{r}}{L}\left.\bar{\bar{T}}\right|_{x=L} \tag{A5}$$

where

$$\bar{\bar{T}} = \int_0^\infty e^{-pt}\tilde{T}(x, \mu_i, t)\,dt, \qquad \text{and} \qquad \bar{f}(p) = \int_0^\infty e^{-pt} f(t)\,dt$$

The solution of Eq. (A4) for $x = L$, with respect to conditions given by Eq. (A5), is

$$\bar{\bar{T}}(L) = T_0 \frac{L^2}{\alpha} \bar{f}(p) \frac{\beta_i \tilde{g}(\mu_i)}{(\beta_i^2 + H_\mathrm{f} H_\mathrm{r}) \sinh(\beta_i) + \beta_i (H_\mathrm{f} + H_\mathrm{r}) \cosh(\beta_i)} \tag{A6}$$

where β_i is given by Eq. (12).

By inverting the Hankel transform in Eq. (A6), using the known formula

$$\bar{T}(L, 0, p) = \sum_{i=1}^{\infty} K(\mu_i, 0)\,\bar{\bar{T}}(L) \tag{A7}$$

we obtain the form for $\bar{T}(L, 0, p)$ given by Eq. (18).

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REFERENCES

1. W. J. Parker, R. J. Jenkins, C. P. Butler, and G. L. Abbott, *J. Appl. Phys.* **32**:1679 (1961).
2. Y. Iida, T. Takashima, and H. Shigeta, *Proceedings of the 2nd Japan Symposium on Thermophysical Properties* (1981), p. 1 (abstract in English).
3. Y. Iida, H. Ohura, and T. Muraoka, *Proceedings of the 6th Japan Symposium on Thermophysical Properties* (1985), p. 85 (abstract in English).
4. Y. Iida and M. Kikuchi, *Proceedings of the 9th Japan Symposium on Thermophysical Properties* (1988), p. 25 (abstract in English).

5. Ph. Delpech, D. Boscher, A. Deom, and D. Balageas, *Actes du Colloque de Thermique PROGRES et DEFIS ACTUELS, Vol. 2*, ISITEM, Nantes, 16–17 May 1990 (SFT and ISITEM, Nantes, 1990), p. 177 (in French).
6. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed. (Oxford University Press, Oxford, 1959), p. 298.
7. K. B. Larson and K. Koyama, *J. Appl. Phys.* **38**:465 (1967).
8. K. B. Larson and K. Koyama, *J. Appl. Phys.* **39**:4408 (1968).
9. L. I. Turchak, *Basics of Numerical Methods* (NAUKA, Moscow, 1987), p. 99 (in Russian).
10. J. Gembarovič, L. Vozár, and V. Majerník, *Int. J. Heat Mass Transfer* **33**:1563 (1990).
11. T. Azumi and Y. Takahashi, *Rev. Sci. Instrum.* **52**:1411 (1981).
12. R. E. Taylor and L. M. Clark III, *High Temp. High Press.* **6**:65 (1974).